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Solution by G. B. M. ZERR, A. M., Ph. D.

Let $u=\int_0^\infty \frac{\sin 2nx\,dx}{(a^2+x^2)\sin x}$, then

$$\frac{du}{dn}=\int_0^\infty \frac{2x\cos nx dx}{(a^2-x^2)\sin x},\quad \frac{d^2u}{dn^2}=-\int_0^\infty \frac{4x^2\sin 2nx dx}{(a^2+x^2)\sin x}=4a^2u-4\int_0^\infty \frac{\sin 2nx dx}{\sin x}.$$

$$\text{Let } v=8\int_0^\infty \frac{\sin nx\cos nx\,dx}{\sin x}=8\int_0^\infty \frac{e^{nx\sqrt{-1}}-e^{-nx\sqrt{-1}}}{e^{x\sqrt{-1}}-e^{-x\sqrt{-1}}}\cos nx dx.$$

Let $x\sqrt{(-1)}=\pi y$, $dx=-\pi\sqrt{(-1)}dy$;

$$\therefore v=-8\pi\sqrt{(-1)}\int_0^\infty \frac{e^{\pi ny}-e^{-\pi ny}}{e^{\pi y}-e^{-\pi y}}\cos(\pi ny\sqrt{-1})dy$$

$$=-8\pi\sqrt{(-1)}\left(\frac{\sin\pi n}{e^{\pi n\sqrt{-1}}+2\cos\pi n+e^{-\pi n\sqrt{-1}}}\right)=-2\pi\sqrt{(-1)}\tan\pi n.$$

$\therefore v=0$, since $n=$any integer. $\therefore \frac{d^2u}{dn^2}-4a^2u=0$.

$u=Ae^{2an}+Be^{-2an}$. When $n=0$, $u=0$, and $B=-A$. $\therefore u=A(e^{2an}-e^{-2an})$.

When $n=1$, $u=\frac{\pi e^{-a}}{a}$ and $A=\frac{\pi e^{-a}}{a(e^{2a}-e^{-2a})}$.

$$\therefore u=\frac{\pi e^{-a}}{a}\cdot\frac{e^{2an}-e^{-2an}}{e^{2a}-e^{-2a}}=\frac{\pi e^{-a}}{a}\cdot\frac{\sinh 2an}{\sinh 2a}.$$

184. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

If $u=f(x,y)$; $\xi=e^xy$; $y\eta=e^x$; show that

$$\frac{d^2u}{dx^2}-y^2\frac{d^2u}{dy^2}-y\frac{du}{dy}=4\xi\eta\frac{d^2u}{d\xi.d\eta}.$$

Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

We have $x=\frac{1}{2}\log\xi+\frac{1}{2}\log\eta$, $y=\sqrt{(\xi/\eta)}$, and therefore

$$\frac{\partial}{\partial\xi}=\frac{\partial x}{\partial\xi}.\frac{\partial}{\partial x}+\frac{\partial y}{d\xi}.\frac{\partial}{\partial y}=\tfrac{1}{2}\left(\frac{1}{e^x y}\frac{\partial}{\partial x}+\frac{1}{e^x}\frac{\partial}{\partial y}\right),\text{ whence } 2\xi\frac{\partial}{\partial\xi}=\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}.$$

In a similar way we get

$$2\eta\frac{\partial}{\partial\eta}=\frac{\partial}{\partial x}=y\frac{\partial}{\partial y}.$$

$$\therefore 4\xi\eta\frac{\partial^2 u}{\partial\xi\,\partial\eta}=\left(\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x}-y\frac{\partial}{\partial y}\right)u=\frac{\partial^2 u}{\partial x^2}-y\frac{\partial^2 u}{\partial x\,\partial y}+y\frac{\partial^2 u}{\partial x\,\partial y}$$

$$-y\frac{\partial u}{\partial y}-y^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}-y\frac{\partial u}{\partial y}.$$

Also solved by G. B. M. Zerr.

MECHANICS.

170. Proposed by **ELISHA S. LOOMIS, Ph. D., Berea, Ohio.**

Two angles of iron, A_1CD and A_1CA_3, move freely on a pivot at C. Rods B_1A_1 and B_1A_3 are attached, respectively, at A_1 and at some point A_3 so that when B_1 moves along the rod CR, which is perpendicular to A_1A_4, CD and CA_3 shall coincide in position with CE which is perpendicular to rod KR. When angle A_1CD is 135° find CA_3 in terms of CA_1. Also find the following:

a) That value of CB_1 which will require least effort exerted at B_1 to cause CA_3 to take the position CA_4.

b) That value of CB_1 which will cause B_2A_2, if produced, to pass through the point A_1.

c) As CB_1 varies in value, what is the locus of the intersection of A_1B_1 and A_2B_2? Of B_1A_3 and B_2A_4?

d) Suppose angle A_1CD to be any other angle than 135°, then find CA_3 in terms of CA_1.

(For figure see pp. 177 and 212.)

II.. Solution by **G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let $CB_1=a$, $CA_1=r$, $CA_3=x$, $\angle A_1CA_2=\angle A_3CA_4=\beta$.

$\therefore A_1B_1=\sqrt{(a^2+r^2)}$.

d) $A_2B_2=A_1B_1=\sqrt{(a^2+r^2)}=\sqrt{(r^2+CB_2^2-2r.CB_2\sin\beta)}$.

$\therefore CB_2=r\sin\beta\pm\sqrt{(a^2+r^2\sin^2\beta)}$.

$A_4B_2=x^2+2r^2\sin^2\beta+a^2+2r\sin\beta\sqrt{(a^2+r^2\sin^2\beta)}$.

$A_3B_1=A_4B_2=x^2+a^2+2ax\sin\beta$.

$$\therefore x=\frac{r^2\sin\beta+r(a^2+r^2\sin^2\beta)}{a}.$$

When $\beta=\frac{1}{4}\pi$, $x=\dfrac{r\sqrt{2}\,[r+\sqrt{(2a^2+r^2)}]}{2a}$.

a) Let P=pull along A_3B_1, Q=resolved part of pull along A_3C.

Then $Q=P\cos CA_3B_1=P\cos\theta$

$$=\frac{P[2x+\sqrt{(2)}a}{2\sqrt{[x^2+a^2+ax\sqrt{2}]}}=\frac{Pr[r+a\sqrt{2}+\sqrt{(2a^2+r^2)}]}{[r+\sqrt{(2a^2+r^2)}]\sqrt{(a^2+r^2)}}$$

$$=\frac{Pr}{\sqrt{(a^2+r^2)}}+\frac{Pra\sqrt{2}}{[r+\sqrt{(2a^2+r^2)}]\sqrt{(a^2+r^2)}}=a \text{ minimum.}$$